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First United Kingdom Doppler campaign: results and interpretation

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The first part of the United Kingdom Doppler campaign was carried out in 1976 and involved 13 stations of the primary triangulation network of Great Britain. These were supplemented in 1978 by observations carried out by Decca Survey Limited on seven additional stations, thus covering the whole of the OSGB Network.

The observations were processed in the single point positioning mode with both the broadcast ephemeris and the precise ephemeris. All of the data were processed with the use of programs developed at Nottingham. Moreover, and for the sake of comparison, a small sample of the data was also run with several commercially available programs. The analysis of the results indicated some significant discrepancies between the geocentric coordinates obtained by using the two types of ephemeris. These differences were confirmed by similar comparisons carried out on a continental scale.

The paper is concluded with practical suggestions aimed at helping organizations processing Doppler data with only the broadcast ephemeris.

1. Introduction

As the first stage of a proposed investigation into absolute and relative accuracies in terrestrial positioning by satellite-Doppler methods (Ashkenazi & Richards 1976), a satellite-Doppler observational campaign was conducted in the United Kingdom. The observations were carried out in 1976 by a team of surveyors from the Ordnance Survey of Great Britain working in cooperation with the University of Nottingham. The data was processed with the UNDAP program developed at Nottingham, with the use of both the broadcast and the precise ephemerides. The results were presented in terms of datum shift parameters for converting Doppler derived coordinates to the OSGB 70 network. For the sake of comparison a small sample of the data was also run with commercial programs developed by Marconi, JMR and DBA.

The analysis of the results pointed to some significant discrepancies between the geocentric coordinates obtained by using the two types of ephemeris. In order to achieve coverage of the whole of the United Kingdom, the data were supplemented in 1978 by observations carried out by Decca Survey Ltd.

Following private communications with a member of the Johns Hopkins Applied Physics Laboratory (APL/JHU) after the publication of the preliminary results at a JLG meeting in Luxembourg in December 1977, the observations from those stations, which were part of the 1977 readjustment of the OSGB Scientific Network, were reprocessed. These results, supplemented by additional data from the second stage of the U.K. Doppler Campaign (Ashkenazi

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et al. 1977) and earlier data from the 512 Specialist Team Royal Engineers (512 S.T.R.E.), were used to derive transformation parameters between coordinates computed by using the two satellite reference systems and the OSGB 77 values. The differences between the two satellite ephemeris systems are highlighted in the paper, which is concluded by practical suggestions to overcome the difficulties presented.

2. Doppler transformation parameters

The definition of a classical geodetic reference system involves the choice of eight independent constants: two for the direction of the minor axis of the reference ellipsoid, two for its size and flattening, three for the position of the centre of the spheroid and one defining the zero geodetic longitude (Bomford 1971). Each reference ellipsoid has an associated Cartesian coordinate system, with the origin at the centre of the spheroid, the Z-axis in the direction of the CIO pole, the X-axis in the direction of BIH zero longitude and the Y-axis perpendicular to these two, to form a right-handed system. It follows that the X (as well as the Y and Z) axes of all such properly defined systems are parallel among themselves and transformation from one system to another can be achieved by using changes in the two ellipsoidal parameters, Δa and Δf (semi-major axis and flattening) and three translation parameters, ΔX, ΔY and ΔZ, but no *rotations*.

However, the reference systems used for the broadcast and precise satellite ephemerides are based on the allocation of well considered, but nevertheless arbitrary, geodetic coordinate values to a number of ground stations belonging to Opnet and Tranet, respectively. The Doppler reference systems are therefore not properly defined and may not be directly connected to other classical geodetic systems. However, the resulting pole position, zero longitude and scale biases have been very well investigated through painstaking comparisons with other global systems involving different Earth models. As a result, a series of small corrections have been computed which, if applied to satellite-Doppler derived coordinates, turn these into Cartesian and corresponding spheroidal geodetic coordinates which may be considered to be properly defined.

To perform a general transformation from a Doppler coordinate system to a terrestrial network, up to seven parameters may be used, namely, one for the scale correction (μ), three for translation (ΔX, ΔY and ΔZ) and three for rotation (θ_x, θ_y and θ_z). These parameters may be derived from a least-squares Helmert transformation in which the 'observed' quantities are the differences at several stations between the Doppler derived Cartesian coordinates and the values obtained by classical methods in the terrestrial system. One can use fewer than seven transformation parameters, especially when the values of some of these can be determined externally by some other means. The transformations discussed in §4 were derived by using models of 2, 3 (translation only), 3 (one each of translation, rotation and scale), 4, 5 and 7 parameters. They are accompanied by their *a posteriori* standard errors and variance factors. The specific mathematical model used in the computation of the Helmert transformation is attributed to Bursa and is described in detail by Leick & van Gelder (1975).

3. Selection of stations

During the first stage of the campaign, 13 stations were occupied in succession, with the use of a Marconi CMA-722B receiver. Three of the stations were also observed with a JMR-1. All of these stations were pillars either forming part of the OSGB Scientific Network or linked to it by connections of a primary geodetic standard. The sites were chosen in clusters of two or three so that any systematic errors in the terrestrial network could be distinguished from observational or ephemeral errors in the Doppler data (Ashkenazi *et al.* 1977). These stations, which were observed in 1976 (UK 76), were supplemented by observations made in 1978 by Decca Survey at seven primary triangulation sites in Scotland and Wales by using JMR-1 receivers.

The second stage of the campaign (UK 78) involved the occupation of nine stations on the Dover–Malvern–Edinburgh precise traverse (Ashkenazi & Sykes, 1978, this symposium). After an analysis of the quality of the various Doppler point positions, it was decided to include seven UK 78, three UK 76 and three Decca observed stations in computations involving the broadcast ephemeris. For data processed with the precise ephemeris, observations from nine UK 78, four UK 76, four Decca and the seven 512 S.T.R.E. stations were included.

4. Results and interpretation

4.1. *UK* 76 *results*

The results of the first stage of the campaign were presented in terms of the three translation parameters, ΔX, ΔY and ΔZ, between the origin of the Doppler Cartesian coordinate system and the centre of the OSGB 70 reference ellipsoid. The Doppler coordinates were obtained either directly from the broadcast ephemeris (which at the time was thought to be with reference to the WGS 72 system) or from the precise ephemeris on the NWL-9D datum corrected for longitude and scale biases, thus converting them into the NWL-10F system. As of 15 June 1977, the NWL-9D system has been replaced by the NSWC 9Z-2 system with a different gravity field. This change does not affect users. The corrections used then (1976) were those given by Anderle (1975), namely $+0.26''$ for longitude rotation and -5.27 m in the radius.

After being processed, the Doppler data collected by Nottingham's CMA-722B were found to be of poor quality, because of a fault in the receiver. This meant that the amount of useful data collected was seriously reduced and this was especially critical when the data were processed in conjunction with the precise ephemeris. However, despite the small number of passes available from each site, the individual translation parameters derived at the stations showed good agreement. Furthermore, no systematic variation over the United Kingdom could be detected and the results produced by the commercial programs were consistent with those from UNDAP. The following mean translation parameters were derived (Ashkenazi *et al.* 1977):

broadcast ephemeris to OSGB 70

$$\Delta X = -371\ \text{m};\quad \Delta Y = +126\ \text{m};\quad \Delta Z = -440\ \text{m}$$

precise ephemeris (corrected) to OSGB 70

$$\Delta X = -369\ \text{m};\quad \Delta Y = +122\ \text{m};\quad \Delta Z = -431\ \text{m}.$$

Clearly, the discrepancies between the two Doppler systems of -2 m in X, $+4$ m in Y, and -9 m in Z, required an explanation.

Comparisons with values obtained by other researchers seemed largely to confirm the substance of the Nottingham findings. However, a private communication from R. E. Jenkins (1978) pointed out that there were two sources of misunderstanding in the analysis: (*a*) The broadcast ephemeris computed coordinates should not be referred to the WGS 72 but to the

NWL-10D system which is almost compatible with the NWL-9D system; (*b*) The value of the Earth rotation rate should be $\omega = 0.000\,072\,921\,158\,55$ rad/s, rather than the value published by T. O. Seppelin (1974).

As a result of this exchange it was decided to compare the broadcast and uncorrected precise ephemeris results for 13 stations in the UK 76 campaign. The resulting three-parameter transformations were found to be +1 m in X, −1 m in Y and −5 m in Z. It was also decided to attempt the derivation of these transformation parameters between the two systems over a much larger area, by using the preliminary Edoc-2 results (observed throughout Europe). The resulting significant transformations were +3 m in X and −5 m in Z, neglecting the scale parameter, and −3 m in X, −11 m in Z and a scale factor of +1.3 parts/10^6 with the scale parameter included (Ashkenazi & Sykes 1978).

4.2. *Broadcast to precise ephemeris transformation*

All of the data previously mentioned in §3 were also used to derive the transformation parameters between the two ephemeris systems (table 1).

The results indicate that the only significant parameters are a shift of −1 m in Y and −4 m in Z if the scale parameter is neglected. These values agree reasonably well with those obtained previously (see §4.1). Moreover, considering the size of the OSGB network and the accuracy of positions determined with the broadcast ephemeris, it is very difficult to determine a small-scale discrepancy between the two systems. This is shown by the apparently exaggerated values produced by the models involving 4, 5 and 7 parameters. A further difficulty arises from the fact that a rotation about the Z axis cannot be distinguished from a shift in the Y coordinate for a small area such as the United Kingdom, straddling the Greenwich meridian.

For these reasons, it was decided to adopt the values computed over the much larger continental area of Europe (Ashkenazi & Sykes 1978). These include a scale bias between the two ephemeris systems of +1.3 parts/10^6 and an origin shift of −3 m in X and −11 m in Z. It is essential to use this set of transformation parameters rather than one of just three translations because otherwise any distance computed between the resulting Doppler positions will be in error by 1.3 parts/10^6 (Ashkenazi & Sykes 1978, this symposium).

4.3. *Transformation from precise ephemeris to OSGB* 77

The precise ephemeris satellite coordinates and thus the Doppler derived positions of the tracking stations are in the NWL-9D reference system. Various sets of values have been quoted in recent years for the scale and longitude corrections which convert this system into a proper geodetic (CIO pole and BIH zero longitude based) reference system (Anderle 1975). The most recently proposed values of these parameters are +0.8″ in longitude and −0.4 part/10^6 in scale (Anderle 1978; Hothem *et al.* 1978).

Transformation parameters between the Doppler and the OSGB systems were computed for both the (uncorrected) NWL-9D values of the Doppler positions as well as for the (corrected) Doppler 78 system of coordinates. These are listed in table 2 and table 3 respectively, together with their *a posteriori* standard errors and the corresponding variance factors.

It should be mentioned that the OSGB terrestrial coordinates used in these comparisons were also the latest available, namely OSGB 77 values.

An examination of the transformations from Doppler 78 to OSGB 77 (table 3) show that the models involving 5 and 7 parameters are not valid, because they include both a Z-rotation

Table 1. Broadcast to precise ephemeris transformation parameters over the U.K.

number of parameters	ΔX/m	ΔY/m	ΔZ/m	$10^6\,\mu$	θ_x	θ_y	θ_z	unit variance
2	—	—	—	-0.5 ± 0.1	—	—	$+0.1 \pm 0.1''$	5.55
3	—	—	-3.9 ± 0.9	$+0.0 \pm 0.1$	—	—	$+0.1 \pm 0.1''$	3.75
3	$+0.1 \pm 0.5$	-1.1 ± 0.5	-3.7 ± 0.5	—	—	—	—	3.75
4	$+16.3 \pm 7.8$	-1.7 ± 0.6	$+17.8 \pm 10.4$	$+4.2 \pm 2.0$	—	—	—	3.42
5	$+16.1 \pm 7.9$	-8.5 ± 9.2	$+17.8 \pm 10.5$	-4.2 ± 2.1	—	—	$-0.4 \pm 0.5''$	3.46
7	-18.1 ± 18.2	$+63.2 \pm 42.0$	$+45.4 \pm 16.7$	-5.2 ± 2.0	$-1.9 \pm 1.1''$	$-1.4 \pm 0.7''$	$+1.0 \pm 0.9''$	3.24

Table 2. Uncorrected precise to OSGB 77 transformation parameters over the U.K.

number of parameters	ΔX/m	ΔY/m	ΔZ/m	$10^6\,\mu$	θ_x	θ_y	θ_z	unit variance
3	-370.3 ± 0.3	$+127.0 \pm 0.3$	-432.5 ± 0.3	—	—	—	—	2.78
4	-354.1 ± 4.2	$+126.3 \pm 0.4$	-410.9 ± 5.6	-4.2 ± 1.1	—	—	—	2.30
5	-354.6 ± 4.1	$+116.0 \pm 4.7$	-410.9 ± 5.4	-4.2 ± 1.1	—	—	$-0.6 \pm 0.3''$	2.18
7	-344.6 ± 7.9	$+59.8 \pm 17.9$	-420.2 ± 7.5	-4.2 ± 1.0	$+1.6 \pm 0.5''$	$+0.4 \pm 0.3''$	$-1.5 \pm 0.4''$	1.93

Table 3. Corrected precise (Doppler 78) to OSGB 77 transformation parameters over the U.K.

number of parameters	ΔX/m	ΔY/m	ΔZ/m	$10^6\,\mu$	θ_x	θ_y	θ_z	unit variance
3	-369.4 ± 0.3	$+112.1 \pm 0.3$	-430.5 ± 0.3	—	—	—	—	2.57
4	-354.8 ± 4.0	$+111.5 \pm 0.4$	-410.9 ± 5.4	-3.8 ± 1.1	—	—	—	2.18
5	-354.6 ± 4.1	$+116.0 \pm 4.7$	-410.9 ± 5.4	-3.8 ± 1.1	—	—	$-0.2 \pm 0.3''$	2.19
7	-344.6 ± 7.9	$+59.8 \pm 17.9$	-420.2 ± 7.5	-3.8 ± 1.0	$+1.6 \pm 0.5''$	$+0.4 \pm 0.3''$	$-0.7 \pm 0.4''$	1.93

and a Y-translation simultaneously. Moreover, considering the size of the OSGB network (approximately 1000 km along its longest dimension) and the accuracy of a Doppler fix with the precise ephemeris (± 1 m), clearly one can determine a scale difference to no better than 1 part/10^6. This makes the resulting value of 3.8 parts/10^6 very plausible, because it also agrees with other external evidence showing a possible scale error of 2.6 parts/10^6 in the terrestrial distances related to the OSGB 77 network.

This leaves the practical Doppler user in the United Kingdom with a dilemma. He may choose to maintain the 'correct' absolute scale in transferring his Doppler coordinates to the OSGB 77 system, in which case he should apply three translation parameters of $-369, +112$ and -431 m. Alternatively, he may decide to replace the 'correct' scale with that of the OSGB 77 network, for even greater conformity with it, in which case he should use three other translation parameters of -355, $+112$ and -411 m *and* a scale factor of -3.8 parts/10^6.

Clearly the decision will depend to a large extent on the specifications of the job in hand. If the specifications strictly state that complete compatibility with the OSGB 77 system is necessary, then the latter method should be adopted. Failing this, clearly the former model is more satisfactory.

5. Conclusions

(1) If the broadcast ephemeris is used in processing the data for individual point positioning, then the resulting Cartesian coordinates must be transformed into the NWL-9D system by applying some empirical correction parameters.

(2) In Europe (and the United Kingdom), the recommended values of these corrections are -3 m in X, -11 m in Z and 1.3 parts/10^6.

(3) The transformation tests appear to indicate that one could use only translation parameters (in this case $+3$ m in X and -5 m in Z) without applying any scale correction. However, further tests at Nottingham University strongly support the adoption of the transformation model in (2) above, *with* a scale parameter.

(4) Doppler coordinates in the NWL-9D system should be transformed into the latest Doppler reference system (Doppler 78) by applying the corrections of $+0.8''$ in longitude and -0.4 part/10^6 in scale.

(5) The recommended parameters for transforming Doppler 78 coordinates into the OSGB 77 terrestrial system (with its 3 parts/10^6 error) are translations of -355 m in X, $+112$ m in Y, -411 m in Z and a scale correction of -3.8 parts/10^6.

(6) The recommended parameters for transforming Doppler 78 coordinates into a terrestrial system based on OSGB 77 but with 'correct' scale are translations of -369 m in X, $+112$ m in Y and -431 m in Z.

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References (Ashkenazi *et al.*)

Anderle, R. J. 1975 Error model for geodetic positions derived from Doppler satellite observations. *NSWC/DL tech. Rep.* no. 3368.

Anderle, R. J. 1978 Mean Earth ellipsoid based on Doppler satellite observations. Prepared for the Spring Meeting of the American Geophysical Union, Florida, April 1978.

Ashkenazi, V. & Richards, M. R. 1976 A proposed investigation on absolute and relative accuracies in terrestrial positioning by satellite-Doppler methods. *Chart. Surv.* **3** (4), 49–50.

Ashkenazi, V., Gough, R. J. & Sykes, R. M. 1977 Preliminary analysis of the United Kingdom Doppler campaign. Presented at 35th J.L.G. Meeting, Luxemburg.

Ashkenazi, V. & Sykes, R. M. 1978 Precise and broadcast emphemerides position determination over the U.K. and Europe. Presented at 2nd Int. Symp. on the Use of Artificial Satellites in Geodesy and Geodynamics, Athens.

Bomford, G. 1971 *Geodesy*, 3rd edn. Oxford: Clarendon Press.

Hothem, L. D., Robertson, D. S. & Strange, W. E. 1978 Orientation and scale of satellite-Doppler results based on combination and comparison with other space systems. Presented at the Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks, Washington, April 1978.

Leick, A. & Van Gelder, B. H. W. 1975 On similarity transformations and geodetic network distortions based on Doppler satellite observations. *Dept of Geodetic Science*, *Ohio State University*, *Report* no. 235.

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